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Independent component analysis of locally fixed on the blind signal separation algorithm with unknown source number

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Abstract. In order to improve the accuracy and robustness of blind signal separation with unknown source number. In this paper, a blind signal separation algorithm based on uniform circular array is proposed, which uses the amplitude of the source signal is independent of the statistics and spatial position distribution independence. The algorithm first uses independent component analysis to estimate the separation matrix of the source signal, then uses the space spectrum of uniform circular array to modify it, and finally use the modified separation matrix to separate the source signals. It is proved by simulation experiments that the method presented in this paper is robust and the algorithm of this paper is still valid.

Key words. Blind signal separation, independent component analysis, uniform circular array, unknown source number.

1. Introduction

Blind signal separation (BSS) is a technology which aims to separate unknown source signal from the received mixed array. BBS can be used to separate the voice of every person without knowing channel parameters. BSS is applied to communi-

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cations, radar and direction of arrival estimation[1,2,3]. When conducting BSS of unknown source number, Xu assumes that the BSS output signal number is equal to the sensors. In the case there is no noise, the source signal and several redundant zero signals will be separated [4]. For mixed signals with noises, the norm of separative matrix tends to be infinite through numerous iterations, so the algorithm is rather unstable. Amari et al. take a method to extract a certain number of source signals [5, 6, and 7]. If the number is smaller than or equal to the signal number of the true source, then the extracted reparative signal will be an estimation of several or all source signals. If the number is larger than the source signal number the true, some redundant zero signals will be generated. In case of noises for the mixed signal, the residual of reparative signal will accumulate mutually, thus affecting the separation effect. The redundant signal is also not with zero range [8, 9, and 10].

To improve the robustness and accuracy of BSS of unknown source number, this paper combines with the independent component analysis and spatial spectrum estimation method. Based on the statistical independence of signal amplitude and spatial distribution independence of signal, it is to amend the independent component analysis by making use of spatial spectrum, thus making the blind signal separation more accurate and steadier. Finally, the effectiveness of the algorithm is verified through the simulation experiment.

2. Problem description of BSS

In order to use spatial information to correct the blind signal separation, we specify the sensor array as a uniform circular. This design is feasible in practical applications, such as the radar antenna can be designed for uniform circular array. Use (1) to describe the observation signal of array element.

$$\tau = \tau_0 \left(1/2 - \xi\right) , \tag{1}$$

Where $x(k) = [x_1(k), x_2(k), ..., x_M(k)]^H$is dada snapshot of $M \times 1$dimension; $s(k)$is the $N \times 1$dimension vector of spatial signal; Where $x(k) = [x_1(k), x_2(k), ..., x_M(k)]^H$is dada snapshot of $M \times 1$dimension; $s(k)$is the $N \times 1$dimension vector of spatial signal;

$$x(t) = \sum_{i=1}^{N} a(\theta_i) s_i(t) + n(t)$$

$$= AS(t) + n(t)$$

$$(where, A = [a_1, a_2, ..., a_N]) \tag{2}$$

Is the direction vector array of $M \times N$dimension;

$$a_i = [\exp(-j2\pi f_c \tau_{1i}), \exp(-j2\pi f_c \tau_{2i}), ..., \exp(-j2\pi f_c \tau_{Mi})]^H \tag{3}$$

Is direction vector; $\tau_{ki} = \frac{r}{c} \cos(\frac{2\pi(k-1)}{M} - \theta_i) \cos\varphi_i$is delay. Here, Under the circumstance that A and the source signal are unknown in the separation system of (1), the separation matrix W is determined only by using the observation signal X(k), so

$$y(k) = Wx(k) \tag{4}$$

Is the estimation of source signals$s(k) = [s_1(k), s_2(k), ...s_n(k)]^H$.

3. BSS based on independent component analysis (ICA)

A and the source signal are unknown, so the direct calculation of formula (4) is unrealistic. Therefore, by looking for the separative matrix W, it is to make components of y(k) independent as far as possible. In this case, y(k) will be the estimation of s(k), thus reaching the purpose of BSS. This is the basic idea of BSS based on ICA.

There are basic hypotheses shown below for BSS algorithm with independent component analysis:

Hypothesis 1: Components of s(k) are statistically independent;

Hypothesis 2: One component at most shows Gaussian distribution.

The cost function that is used to measure the independence can be the probability density function based on the priori or estimated source signal or that based on the nonlinear function in accordance with the method proposed by [8]. If the probability density function of the source signal can be estimated, or the nonlinear function is close to the probability density function of the actual source signal, then the ICA algorithm will be with very good astringency, then the source signal will be separated or the mixed matrix A can be identified[9]. However, in practical application, the probability density function of the source signal is unknown in most cases, so it is hard to estimate or replace with nonlinear functions. The cost function based on the four-order statistics is selected by this paper[7].

$$E = E_0 \left(1 - \gamma\tau\right) , \tag{5}$$

Where,

$$h(\xi) = h_0 \left[1 - \left(1 - \beta_1\right)\left(\xi + 1/2\right)\right] \cdot \left[1 - \left(1 - \beta_2\right)\left(\eta + 1/2\right)\right] , \tag{6}$$

In case there is noise, the robustness algorithm of formula (5) is

$$\rho = \rho_0 \left[1 - \left(1 - \beta\right)\left(\xi + 1/2\right)^2\right] , \tag{7}$$

Where,

$$T = \frac{ab}{2}\omega^2 \int_A h(\xi)\rho w^2 \,\mathrm{d}A \tag{8}$$

$$V = \frac{ab}{2} \int_A D(\xi)[G - 2(1-\nu)H] \,\mathrm{d}A , \tag{9}$$

Obviously, in case of convergence, T=0,F=0 and formula (14) are the same as the formula (5). However, formula (7) shows better robustness of noises.

After the iterative convergence of formula (7), the separative signal can be obtained via the formula (4). The separative signal will be the estimation of the source signal.

In case the source number is unknown, the value e cannot be determined, so it may lead to the leakage of separation, and the separation accuracy at low signal to noise ratio is not high.

4. The source signal separation based on spatial spectrum

In addition to the estimation of the direction of arrival (DOA), the spatial spectrum estimation method can also separate the source signal from the observation signal by forming the corresponding beam. For example, by making use of the MUSIC algorithm, DOA can be calculated first, followed by the corresponding direction vector a. Finally, by making use of the minimum variance distortionless response, the beam can be calculated[10].

$$D(\xi) = D_0\left[[1-(1-\beta_1)(\xi+1/2)]\cdot[1-(1-\beta_2)(\eta+1/2)]\right]^3 , \tag{10}$$

Then, the source signal in the corresponding a direction can be separated from the observation signal,

$$D_0 = \frac{Eh_0^3}{12\,(1-\nu^2)} \tag{11}$$

This is the method of separating the source signal by making use of the spatial spectrum. Based on the assumption of this paper, the source number is unknown. When calculating DOA by taking the spatial spectrum method, the lack of estimation or excessive estimation phenomena may occur, which may influence the DOA accuracy, thus affecting the corresponding beam. Finally, the error between the signal separated by formula (11) and the actual source signal may be large.

5. The source signal separation based on spatial spectrum

On the one hand, when separating noises by BSS with the mere use of ICA, the accuracy may be low or unsteady. On the other hand, in case the source signal number is unknown for signal separation based on the spatial spectrum, it may lead to the lack of estimation or excessive estimation. Therefore, the basic idea of this algorithm is to make use of the separative matrix of ICA to calculate the DOA corresponding to different lines of vectors. Next, assuming the source number is different, and the local MUSIC spatial spectrum of the matrix DOA separated by ICA is calculated when the source number differs. When the assumed spatial spectrum peak of the source number has the minimum difference from the corresponding DOA of ICA, then the assumed source number will be the source number of the true time.

Finally, the beam can be formed by making use of the corresponding DOA of the local MUSIC spatial spectrum, and the source signal is separated via the formula (18).

5.1. Calculation of DOA corresponding to the matrix separated by ICA

To modify the accuracy of ICA, the DOA problem corresponding to ICA is discussed in this section first.

Through the iteration of formula (8), e independent source signals can be separated in convergence. At this time, the obtained We is the separative matrix of the independent component, and We is the Moore-Penrose inverse of A, that is

$$D(\xi) = \frac{E_0 h_0^3}{12\,(1-\nu^2)}(p_1 p_2)^3 p_3\,. \tag{12}$$

To calculate the DOA corresponding to lines of We, formula (13)can be used for calculation.

$$T = \frac{ab}{2}\rho_0 h_0 \omega^2 \int_A p_1 p_2 \left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)^2\right] w^2\,\mathrm{d}A\,, \tag{13}$$

W(i,:)is the i-th line vector of the separated matrix, and R is the relevant matrix of the observation signal x(k). The angle corresponding to the spatial spectrum peak obtained by changing A will be the DOA of the i-th line.

Each line vector of the separative matrix We is scanned with O by making use of the formula (20). When the value is the maximum, the corresponding angle will be the DOA of the line vector, that is, the ICA-based DOA estimation can be expressed as

$$V = \frac{ab}{2}\frac{E_0 h_0^3}{12\,(1-\nu^2)}\int_A (p_1 p_2)^3 p_3 (G-2(1-\nu)H)\,\mathrm{d}A \tag{14}$$

The DOA of each separative vector is calculated through formula (14), which can be conducted without knowing the source number. The correct DOA can be estimated by knowing the source number of the MUSIC method.

The estimated DOA is not the final purpose, instead, it is corrected by making use of MUSIC.

5.2. Local fix of the algorithm

To improve the BSS accuracy and robustness of the unknown source number, the ICA separation algorithm is corrected on the basis of section 5.1. First, ICA is used to estimate all ICA-based directions of arrival, which is noted as DOAt . Next, it is assumed that the source number is i=1,2. The source number i is applied to MUSIC for DOAm estimation. When i is the actual source number, the mutual superposition of DOAt and DOAm is more accurate. Therefore, when the value of some i makes the superposition of the ICA-based DOA and MUSIC-based DOA more accurate, the value i should be the value of the source signal number which can be expressed as:

$$\delta\,(V-T)=0\,. \tag{15}$$

In order to find the estimated DOA superposition of the two methods, the following algorithm can be used, which is shown as follows:

1)DOAt is calculated by making use of the formula (14):

$$\eta=\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}\,,\ \eta=-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}\,,\ \xi=-\frac{1}{2}\,,\ \xi=\frac{1}{2}\,. \tag{16}$$

2) The spatial spectrum is obtained and arranged by size

$$w=A_1q_1^2q_2q_3+A_2q_1^3q_2^2q_3^2\,, \tag{17}$$

The corresponding DOA is

$$T=\frac{ab}{2}\rho_0h_0\omega^2\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}p_1p_2\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)^2\right]w^2\,\mathrm{d}\eta\,\mathrm{d}\xi \tag{18}$$

3) Local modification: Assuming the signal space is i, the noise space will be A_1, A_2.The spectrum peak is searched in the local scope a and b by making use of the MUSIC square. The angle of the obtained local spectrum peak is the corresponding DOA of MUSIC.

$$V=\frac{ab}{2}\frac{E_0h_0^3}{12\,(1-\nu^2)}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}(p_1p_2)^3p_3(G-2(1-\nu)H)\,\mathrm{d}\eta\,\mathrm{d}\xi\,. \tag{19}$$

Where

$$\delta\left(V_1-\lambda^2T_1\right)=0\,, \tag{20}$$

It reflects the independent characteristic of the ICA separation vector this time to some extent. If the ICA performance is positive, then C4 is large, and A is small, so the search scope is small. If ICA performance is poor, then C4 is small, and A is large, so the search scope is big.

4) By setting the value i which changes from 1 to k, the formula (18) and (19) are applied to formula (22). In the first time when J(i) is the minimum, DOA will be the DOA obtained by local modification on the ICA basis.

Thus, the modified DOA can be obtained. By making use of the formula (17) and (18), BSS can be carried out.

6. The simulation experiment of the algorithm

To analyze the feasibility of the proposed method and its performance, the following experiment is carried out.

Experiment one: In case SNR differs, the source signal is separated.

Given there are three source signals, namely:

$$q_1 = \left(\xi + \frac{1}{2}\right)\left(\xi - \frac{1}{2}\right), \; q_2 = \eta - \frac{b-c}{2}\xi + \frac{b+c}{4}, \; q_3 = \eta + \frac{b-c}{2}\xi - \frac{b+c}{4},$$

Where, sawtooth(ft) is the sawtooth wave of the frequency f and the central frequency is f=1000Hz. The three signals are incident on the uniform circular array with the incident angle being (300,10), (100,30) and (200,50). The uniform circular array is the six-element array which is isotropic. Signals of each array is collected at the rate of 2000Hz, thus obtaining the observation signal of the array. The separation system is used to separate the observation signal which is the estimation of the source signal.

To observe the performance of the signal separation intuitively, the implicated noise source signal before mixture, the partial observation signal after mixture, the BSS signal based on ICA, and the signal separated by this algorithm are listed and shown in Figure 1.

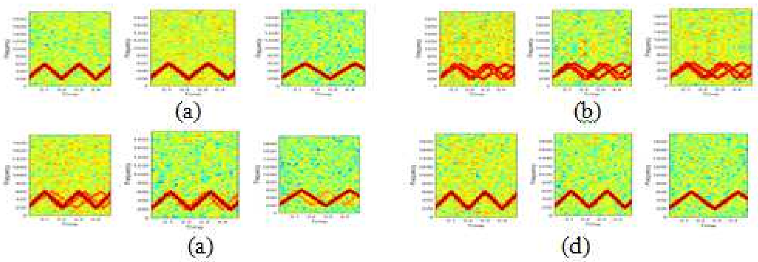

Fig. 1. The time-frequency distribution figure of some bss. (a) The source signal when the signal-to-noise ratio is 10dB; (b) Partial mixed signals; (c) The BSS signal of ICA; (d) The signal separated by this algorithm

As can be seen from Figure 1(c), each component of ICA BSS is with the crosstalk of other sources. Meanwhile, the time-frequency distribution of Figure 1(d) separated signal is the same as that of (a), in which, the source signals can be separated well. Therefore, a better method is proposed by this paper to separate the source signals. To realize the quantitative analysis of the performance of ICA method and the method of this paper, the following performance index (PI) is used to measure the separation effect. PI is defined as

$$T_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}} p_1 p_2 \left[1 - (1-\beta)\left(\xi + \frac{1}{2}\right)^2\right] w^2 \, \mathrm{d}\eta \, \mathrm{d}\xi \,, \tag{21}$$

Where, g is the (i,j)-th element of the global matrix G=WA. The smaller the value

PI is, the better the performance of the separated source signal of the algorithm.

The SNR changes from -10dB to 20dB. 100 times of Monte Carlo source signal separation experiment is done under each SNR value, and the mean value of PI is calculated. The performance index is shown in Figure2.

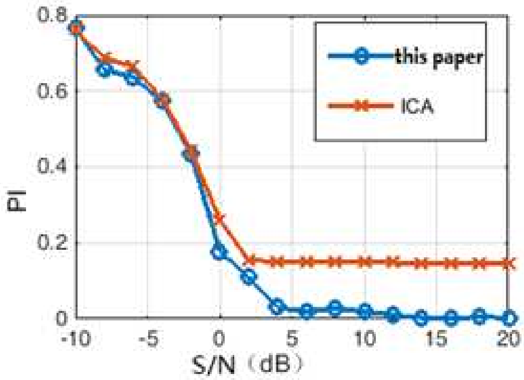

Fig. 2. The performance index of source separation

Seen from Figure2, the performance of the algorithm proposed by this paper is better than the ICA algorithm, especially above 0dB, the PI value of this algorithm is clearly smaller than that of the ICA algorithm.

Experiment two: weak signal separation

The experiment condition includes 6 array elements and 2 incident signals named S1 and S2 (The signal is the same as the experiment one). The incident angle is (300,10) and (100,30) respectively, and the signal-to-noise ratio of S2 is 10dB. S1 is -4dB. The mixed signal is separated by using the ICA and the algorithm of this paper, and the separated results are shown in Figure 3.

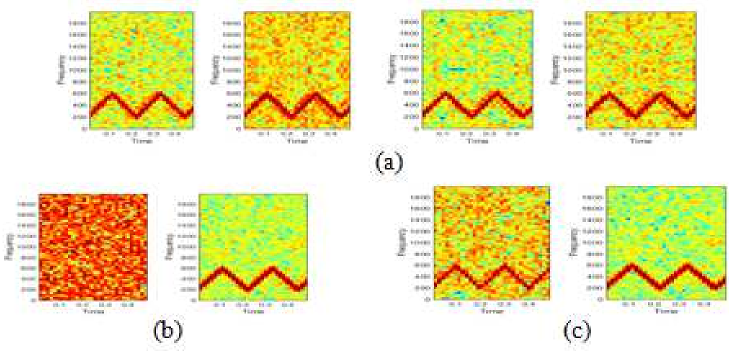

Fig. 3. The time-frequency figure of weak signal separation. (a) Partial mixed signals; (b) The signal separated with ICA algorithm; (c) The signal separated with the algorithm of this paper

Seen from Figure 4(a), the S1 signal in the mixed signal is very weak, and it is almost overwhelmed by S2. S1 cannot be separated by ICA, as shown in Figure

4(b). The algorithm proposed by this paper can separate S1 and S2 successfully, as shown in Figure 4(c). This is because the independence of the source signal and the independence of the spatial distribution are fully utilized by this algorithm. Therefore, the weak source signal can be separated with the algorithm of this paper.

7. Conclusions

A method of calculating the spatial spectrum is proposed by making use of the separative vector of independent component analysis by the algorithm of this paper first, and its rationality is testified. Next, local modification is conducted for the spatial spectrum of separated vectors, and the corrected method is realized via the generation of local spatial spectrum by MUSIC, which is can help to save the storage capacity and reduce the computing time compared to the utilization of the entire space. Finally, the fixed separative vector is used to separate the source signal. In the algorithm proceeding process, the statistical independence characteristic of the source signal amplitude and the distribution independence characteristic of the source signal space position are fully utilized. Through the simulation experiment, it is proved that the source signal number does not have to be estimated in advance for signal separation when conducting blind signal separation of unknown source number, and superior robustness and accuracy can be achieved in contrast to the ICA algorithm. When there is weak source signal in the mixed signal and the ICA is considered noise and cannot be separated, the weak source signal can still be separated with this algorithm.

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